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Cambridge International A Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2024**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

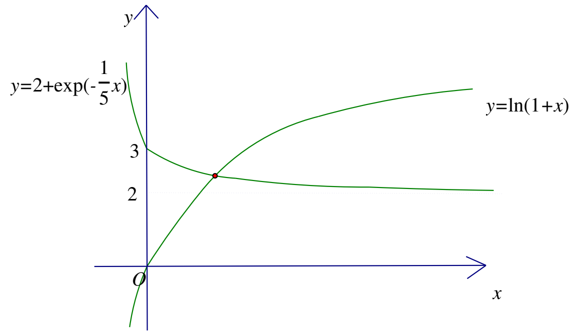
AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Substitute $x = -\frac{1}{2}$ and equate the result to zero	**M1**	
	Obtain a correct equation, e.g. $-\frac{4}{8} + \frac{a}{4} - \frac{5}{2} + b = 0$	**A1**	$\left(\frac{a}{4} + b = 3\right)$ Any equivalent form.
	Substitute $x = 2$ and $x = 4$ and use $\text{p}(4) = 3\text{p}(2)$	**M1**	If using long division, M1 is for correct use of two constant remainders. Condone if 3 is on the wrong side.
	Obtain a correct equation, e.g. $3(32 + 4a + 10 + b) = 256 + 16a + 20 + b$	**A1**	$(-2a + b = 75)$ Any equivalent form.
	Obtain $a = -32$ and $b = 11$	**A1**	
		5	

Question	Answer	Marks	Guidance
2	Integrate to obtain $px^3\ln 3x + q\int x^2\,dx$	**M1***	
	Obtain $\frac{1}{3}x^3\ln 3x - \frac{1}{3}\int x^2\,dx$	**A1**	Or unsimplified equivalent.
	Complete integration to obtain $\frac{1}{3}x^3\ln 3x - \frac{1}{9}x^3$	**A1**	
	Use correct limits correctly in an expression of the form $rx^3\ln 3x + sx^3$	**DM1**	$9\ln 9 - 3 - \frac{1}{3}\ln 3 + \frac{1}{9}$ An exact expression for *their* integral.
	Obtain $\frac{53}{3}\ln 3 - \frac{26}{9}$	**A1**	Or 2-term equivalent.
		5	

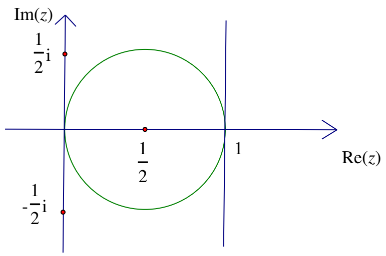
Question	Answer	Marks	Guidance
3	State or imply $\dfrac{1+\frac{dy}{dx}}{x+y}$ as the derivative of $\ln(x+y)$	**B1**	
	State or imply $6xy+3x^2\dfrac{dy}{dx}$ as the derivative of $3x^2y$	**B1**	
	Substitute $(1, 0)$ and solve for $\dfrac{dy}{dx}$	**M1**	Having the correct form for at least one of the above.
	Obtain $\dfrac{dy}{dx}=\dfrac{1}{2}$ or 0.5	**A1**	
	Alternative Method for Question 3:		
	Rewrite as $x+y=e^{3x^2y}$ and state or imply $1+\dfrac{dy}{dx}$ as the derivative of the LHS	**B1**	
	State or imply $\left(6xy+3x^2\dfrac{dy}{dx}\right)e^{3x^2y}$ as the derivative of e^{3x^2y}	**B1**	
	Substitute $(1, 0)$ and solve for $\dfrac{dy}{dx}$	**M1**	Having the correct form for at least one of the above.
	Obtain $\dfrac{dy}{dx}=\dfrac{1}{2}$ or 0.5	**A1**	
		4	

Question	Answer	Marks	Guidance
4(a)	Factorise Or obtain an expression in $\cos^2\theta$ or $\sec^2\theta$	**M1**	$(\sec^2\theta - \tan^2\theta)(\sec^2\theta + \tan^2\theta)$ Or $-1 + \dfrac{2}{\cos^2\theta}$, OE.
	Use of $1 + \tan^2\theta = \sec^2\theta$ (anywhere)	**M1**	The 2 method marks can appear in either order, in which case the 2nd M1 is for expanding $(1+\tan^2\theta)^2 - \tan^4\theta$.
	Obtain $1 \times (1 + \tan^2\theta + \tan^2\theta) = 1 + 2\tan^2\theta$	**A1**	Obtain given answer from full and correct working.
		3	
4(b)	Form an equation in $\tan 2\alpha$. Or multiply through by $\cos^4 2\alpha$ to form an equation in $\sin 2\alpha$ or $\cos 2\alpha$	**M1**	$1 + 2\tan^2 2\alpha = 2\tan^2 2\alpha(1 + \tan^2 2\alpha)$ $\cos^4 2\alpha + 2\sin^2 2\alpha\cos^2 2\alpha = 2\sin^2 2\alpha$ $\Rightarrow \sin^4 2\alpha + 2\sin^2 2\alpha - 1 = 0$ or $\cos^4 2\alpha - 4\cos^2 2\alpha + 2 = 0$
	Solve for $\tan 2\alpha$ or equivalent	**M1**	$\left(\tan 2\alpha = \pm\sqrt{\dfrac{1}{\sqrt{2}}}\right)$
	Obtain one correct solution for α, e.g. $20.0(30..)°$	**A1**	
	Obtain a second correct value for α, e.g. $70.0°\ (69.9698..°)$	**A1**	
	Obtain solutions $110°$ (110.0) and $160°$ (160.0) for α, and no others in range	**A1**	
		5	

Question	Answer	Marks	Guidance
5(a)	Sketch a relevant graph, e.g. $y = 2 + e^{-0.2x}$ For the sketches: Correct curvature Intersections with the y-axis approximately correct Horizontal asymptote approximately correct – need not draw in Allow scale not marked and implied by their sketch	B1	
	Sketch a second relevant graph, e.g. $y = \ln(1+x)$ and justify the given statement	B1	
		2	
5(b)	Calculate the value of a relevant expression or values of a relevant pair of expressions at $x = 7$ and $x = 9$	M1	
	Complete the argument correctly with correct calculated values	A1	E.g. $2.079 < 2.246$ and $2.302 > 2.165$, or $0.167 > 0$ and $-0.137 < 0$.
		2	
5(c)	Use the iterative process correctly at least once	M1	I.e., obtain one value and substitute that value back into the formula.
	Obtain final answer 8.03	A1	
	Show sufficient iterations to at least 4 decimal places to justify 8.03 to 2 decimal places, or show that there is a sign change in the interval $(8.025, 8.035)$	A1	E.g. 7, 8.4555, 7.8846, 8.0849, 8.0115, 8.0380, 8.0283, 8.0318 8, 8.0421, 8.0268, 8.0324, 9, 7.7172, 8.1490, 7.9887, 8.0463, 8.0253, 8.0329
		3	

Question	Answer	Marks	Guidance
6(a)	Correct use of product rule to differentiate	*M1	$2\cos 2x(1+\sin 2x)+\sin 2x \times 2\cos 2x$, or $2\cos^2 x-2\sin^2 x+8\sin x\cos^3 x-8\cos x\sin^3 x$. All terms needed but could have errors in the coefficients.
	Obtain $2\cos 2x+4\cos 2x\sin 2x$	A1	OE
	Equate derivative to zero and solve for $2x$ or x	DM1	$2\cos 2x(1+2\sin 2x)=0 \Rightarrow x=\frac{1}{2}\sin^{-1}\left(-\frac{1}{2}\right)$ Condone if they only consider $\cos 2x=0$.
	Obtain $x=\dfrac{7}{12}\pi,\ y=-\dfrac{1}{4}$	A1	Mark degrees as a misread. The Q asks for an exact answer.
		4	
6(b)	Use correct double angle formula to integrate Or use integration by parts and correct double angle formula	M1	$\int \sin 2x+\dfrac{1-\cos 4x}{2}\,dx$ Or $-\dfrac{1}{2}\cos 2x(1+\sin 2x)+\int\dfrac{1+\cos 4x}{2}\,dx$.
	Obtain $-\dfrac{1}{2}\cos 2x+\dfrac{x}{2}-\dfrac{1}{8}\sin 4x\ (+C)$	A1	Or $-\dfrac{1}{2}\cos 2x-\dfrac{1}{2}\cos 2x\sin 2x+\dfrac{x}{2}+\dfrac{1}{8}\sin 4x\ (+C)$
	Use limits 0 and $\dfrac{1}{2}\pi$ correctly in a solution containing $p\cos 2x$ and $q\sin 4x$	M1	$\dfrac{1}{2}+\dfrac{1}{4}\pi-0+\dfrac{1}{2}-0+0$
	Obtain $\dfrac{1}{4}\pi+1$	A1	
		4	

Question	Answer	Marks	Guidance
7(a)	State or imply the form $\frac{A}{1+2x}+\frac{Bx+C}{2+x^2}$	**B1**	
	Use a correct method to find a constant	**M1**	
	Obtain one of $A=1$, $B=2$ and $C=3$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
7(b)	State $\frac{-1.-2.-3}{3!}(2x)^3$ or -8	**B1 FT**	Correct term in x^3 or coefficient of x^3 in the expansion of $A(1+2x)^{-1}$. Any equivalent form.
	Use a correct method to obtain the coefficient of x^2 in the expansion of $(2+x^2)^{-1}$ or the coefficient of x^2 in the expansion of $\left(1+\frac{x^2}{2}\right)^{-1}$.	**M1**	Do not need to deal with 2^{-1} at this stage.
	Obtain $(Bx+C)\times\frac{1}{2}\times-\frac{1}{2}x^2$ or $-\frac{B}{4}x^3$ or $-\frac{B}{4}$	**A1 FT**	Follow *their B* (and *C*).
	Obtain final answer $-8\frac{1}{2}$ or $-8\frac{1}{2}x^3$	**A1**	Or simplified equivalent. Ignore additional terms for other powers of *x*.
		4	

Question	Answer	Marks	Guidance
8(a)	Multiply numerator and denominator by $1 - y\text{i}$	**M1**	OE
	Obtain $\frac{1}{1+y^2} + \frac{-y}{1+y^2}\text{i}$	**A1**	OE
		2	
8(b)	Express $\left(a - \frac{1}{2}\right)^2 + b^2$ in terms of *y* and expand the bracket	**M1**	$\left(\frac{1}{1+y^2} - \frac{1}{2}\right)^2 + \left(\frac{(-)y}{1+y^2}\right)^2$
	Obtain $\left(\frac{1}{\left(1+y^2\right)^2} - \frac{2}{2\left(1+y^2\right)} + \frac{1}{4}\right) + \frac{y^2}{\left(1+y^2\right)^2}$	**A1FT**	Follow *their* answer from **(a)** provided it gives an expression in *y*.
	Obtain $\frac{1}{4}$ from full and correct working	**A1**	AG
		3	
8(c)	Show a vertical straight line through $1 + 0\text{i}$	**B1**	
	Show a circle centre $\frac{1}{2} + 0\text{i}$	**B1**	
	Show a circle with radius $\frac{1}{2}$ and centre not at the origin	**B1**	
		3	
8(d)	circle centre $\frac{1}{2} + 0\text{i}$ with radius $\frac{1}{2}$	**B1**	OE Condone inclusion of the origin.
		1	

Question	Answer	Marks	Guidance
9(a)	Use a correct method to form a vector equation	**M1**	Allow in column vectors.
	Obtain $\mathbf{r} = 8\mathbf{i} - 5\mathbf{j} + 6\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 4\mathbf{k})$	**A1**	Need **r** = …
		2	
9(b)	State the position vector of a point on l in component form Or at least 2 correct components seen	**B1 FT**	Follow *their* equation $(8+2\lambda)\mathbf{i} + (-5+\lambda)\mathbf{j} + (6+4\lambda)\mathbf{k}$. Might see the correct equation for the first time in **(b).**
	Equate to $-t\mathbf{i} + 4t\mathbf{j} + 3t\mathbf{k}$ and solve for t	**M1**	
	Obtain $t = -2$	**A1**	
		3	
9(c)	Evaluate the scalar product of a pair of relevant vectors	**M1**	$(2\mathbf{i} + \mathbf{j} + 4\mathbf{k}) \cdot (a\mathbf{i} - \mathbf{j} + 3\mathbf{k}) = 2a + 11$ OE, SOI
	Complete the process for finding the cosine of θ	***M1**	Divide the scalar product by the product of the moduli and equate to $\cos\theta$.
	Obtain $\dfrac{2a+11}{\sqrt{21}\sqrt{10+a^2}} = \pm\dfrac{1}{\sqrt{6}}$	**A1**	OE
	Form a 3-term quadratic equation in a and solve for a	**DM1**	$a^2 + 88a + 172 = 0$ OE
	Obtain $a = -2, a = -86$	**A1**	Correct only (both values).
		5	

Question	Answer	Marks	Guidance
10(a)	$\frac{dV}{dt} = \pm k\sqrt{V}$ or $\frac{dV}{dt} = 16\pi\frac{dh}{dt}$	B1	SOI
	Correct use of chain rule and $V = 16\pi h$	M1	OE, e.g. $\frac{dV}{dt} = 16\pi\frac{dh}{dt}$.
	Obtain $\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} = \frac{-k\sqrt{16\pi h}}{16\pi}$	A1	Any equivalent form in terms of *h*.
	$= -\left(\frac{k}{4\sqrt{\pi}}\right)\sqrt{h} = -\lambda\sqrt{h}$ since $\frac{k}{4\sqrt{\pi}}$ is constant	A1	Obtain given answer from full and correct working.
		4	
10(b)	Separate variables correctly and commence integration	*M1	$\int \frac{1}{\sqrt{h}}dh = \int -\lambda dt$ OE
	Obtain $-\lambda t = 2\sqrt{h}(+C)$	A1	
	Use the boundary conditions in an equation containing pt and $q\sqrt{h}$ to form an equation in λ and/or C	DM1	OE, e.g. $0 = 4 + C$ or $-20\lambda = 2\times 1.5 + C$.
	Use the boundary conditions in an equation containing pt and $q\sqrt{h}$ to form a second equation in λ and/or C and solve	DM1	$C = -4$, $\lambda = \frac{1}{20}$ OE, e.g. $-\frac{t}{20} = 2\sqrt{h} - 4$.
	Hence $t = 80 - 40\sqrt{h}$	A1	Must be seen.
	Time to empty the tank is 80 minutes	A1	
		6	